Signs of quadratic function

Part 5

In the previous videos, We’ve learned how to solve a parametric equation of second degree.
In this part, we will learn about parametric example on the signs of roots of a quadratic function.

Parametric example

Consider the quadratic function:

$f(x) = (m+2)x^2 - 2(m+3)x + \text{m} \quad ; \quad \text{m} \neq -2$

You think it is hard!!!

If you follow the given steps, you will find it easy.

Parametric example

Step 1 Calculate the discriminant Δ (or Δ') and study its signs

$$f(x) = (m+2)x^2 - 2(m+3)x + m \quad ; \quad m \neq -2$$

$$a = m+2 \quad ; \quad b = -2(m+3) \quad ; \quad c = m$$

$$\Delta' = b'^2 - ac = (m+3)^2 - (m+2)(m)$$

$$= m^2 + 6m + 9 - m^2 - 2m = 4m + 9$$

$$\Delta' = 0$$
$$4m + 9 = 0$$
$$m = -\frac{9}{4}$$

m	$-\infty$		$-\frac{9}{4}$		-2		$+\infty$
Δ'		$-$	0	$+$	‖	$+$	

Parametric example

Step 2 Calculate the product P and study its signs

$a = m + 2 \quad ; \quad b = -2(m+3) \quad ; \quad c = m$

$P = \frac{c}{a} = \frac{m}{m+2}$

m	$-\infty$		-2		0		$+\infty$
m		$-$		$-$	0	$+$	
$m+2$		$-$		$+$		$+$	
P		$+$		$-$		$+$	

Parametric example

Step 3 Calculate the Sum S and study its signs

$a = m + 2 \quad ; \quad b = -2(m+3) \quad ; \quad c = m$

$$P = \frac{-b}{a} = \frac{2(m+3)}{m+2}$$

m	$-\infty$		-3		-2		$+\infty$
$m+3$		$-$	0	$+$		$+$	
$m+2$		$-$		$-$		$+$	
S		$+$		$-$		$+$	

Parametric example

Step 4 **Draw a general table that summarized all the signs studied before**

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$	‖	$+$		$+$	
P		$+$		$+$		$+$	‖	$-$	0	$+$	
S		$+$	0	$-$		$-$	‖	$+$		$+$	

m	$-\infty$		$-\frac{9}{4}$		-2		$+\infty$
Δ'		$-$	0	$+$	‖	$+$	

m	$-\infty$		-2		0		$+\infty$
m		$-$	‖	$-$	0	$+$	
$m+2$		$-$	‖	$+$		$+$	
P		$+$	‖	$-$		$+$	

m	$-\infty$		-3		-2		$+\infty$
$m+3$		$-$	0	$+$	‖	$+$	
$m+2$		$-$		$-$	‖	$+$	
S		$+$		$-$	‖	$+$	

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots							

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots		$x_1 < x_2 < 0$					

$x_1 < x_2 < 0$

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots		$x_1 < x_2 < 0$		$x_1 < 0 < x_2$ $\lvert x_1 \rvert < x_2$			

$x_1 < 0 < x_2$

$|x_1| < x_2$

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots		$x_1 < x_2 < 0$		$x_1 < 0 < x_2$ $\lvert x_1 \rvert < x_2$		$0 < x_1 < x_2$	

$0 < x_1 < x_2$

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots		$x_1 < x_2 < 0$		$x_1 < 0 < x_2$, $\lvert x_1 \rvert < x_2$		$0 < x_1 < x_2$	

❖ For $m = -3$: the equation is reduced to $-x^2 - 3 = 0$

$$x^2 + 3 = 0$$

So no real roots

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots		$x_1 < x_2 < 0$		$x_1 < 0 < x_2$, $\lvert x_1 \rvert < x_2$		$0 < x_1 < x_2$	

❖ For $m = -3$: the equation is reduced to $-x^2 - 3 = 0$

$$x^2 + 3 = 0$$

So no real roots

❖ For $m = -\frac{9}{4}$: $\Delta' = 0$

So $x_1 = x_2 = -\frac{b}{2a} = -3$

Parametric example

Step 5 Discuss

m	$-\infty$		-3		$-\frac{9}{4}$		-2		0		$+\infty$
Δ		$-$		$-$	0	$+$		$+$		$+$	
P		$+$		$+$		$+$		$-$	0	$+$	
S		$+$	0	$-$		$-$		$+$		$+$	
		No real roots		No real roots		$x_1 < x_2 < 0$		$x_1 < 0 < x_2$, $\lvert x_1 \rvert < x_2$		$0 < x_1 < x_2$	

- For $m = -3$: the equation is reduced to $-x^2 - 3 = 0$

$$x^2 + 3 = 0$$

So no real roots

- For $m = -\frac{9}{4}$: $\Delta' = 0$

So $x_1 = x_2 = -\frac{b}{2a} = -3$

- For $m = 0$: $P = 0$

So $x_1 = 0$; $x_2 = -\frac{b}{a} = +3$

Finally, is it still hard to discuss?